

# Voltage Transient Computation for Transformer Windings with Arresters

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## Abstract

In this paper the resonance of some windings on the transformer is studied. In addition of overvoltage's due to atmospheric discharges, transformers in service are also exposed to ones due change of network states (intermittent short circuits, sudden generator discharges and various manipulations in the network). The character of these overvoltage's is oscillatory, and in the transformer they can produce resonance in the windings or in the parts of windings although the incoming surge amplitude can be less than the protection level of the arrester at the entrance of the plant.

*Keyword: voltage, transient, transformer, computation, ZnO arrester*

## 1. Introduction

The natural frequencies of regulating windings are comparatively low from 10 - 100 KHz, what makes these windings the most sensitive ones. One of most effective ways of protecting transformers against any overvoltage's is to build ZnO arresters in the transformers and to connect them to critical windings. ZnO arresters have markedly non-linear current voltage characteristic (the coefficient of non-linearity ranges from 25 to 75) and high absorbing energy. In the current range from  $10^{-3}$  A to  $10^4$  A, the voltage is changed little and current a great deal, so that its protection factor is better than that of SiC arresters. Besides, unlike SiC arresters, ZnO arresters have no classical spark gaps, and, therefore, they can be very simply mounted in the transformer for correct dimensioning of arresters, the most important data is the quantity of received energy. For this purpose, a calculation of overvoltage's in a transformer with the possibility of setting the arrester can be develop. The calculation method is based on the division of windings into a certain number of elements, that are replaced by concentrated parameters: self-inductances and conductivities. These parameters can be determined from the geometric dimensions of the transformer, and after the matrices are made, i.e. the problem is reduced to solving a system of non-linear

differential equations. Because of markedly non-linear characteristics of ZnO arresters, the system of non-linear differential equations is solved by applying the trapezoidal of integration, which is stable and sufficiently accurate for technical calculations. The incoming surge can be given as the impulse full wave, sinusoidal damped wave or their superposition. A computer programs has been made for the entire calculations procedure. The input data are the ones on transformer windings, the impulse surge and the arresters. The results are the time flows of voltages between windings or parts of windings, the windings and of the one between the windings with regard to the impulse and oscillatory overvoltage's, and dimensioning of ZnO arresters intended for installation in transformers. The results can be represented either in form of tables or graphs. This program enables optimum design of insulation and ZnO arresters for transformers.

## II. DESCRIPTION OF COMPUTATION METHOD

At trapezoidal rule of integration, the network parameters can be replaced by simple equivalent impedance network, that will be explained in the future text.

Inductance for inductance L (Fig.1a) between nodes K and m the equation applies,

$$e_k - e_m = L \frac{di_{k,m}}{dt} \quad (1)$$

And if it integrated from the moment  $t-\Delta t$  with the known state up to the moment  $t$  in which the unknown state is found, it is obtained as follows:

$$i_{k,m}(t) = i_{k,m}(t - \Delta t) + \frac{1}{L} \int_{t-\Delta t}^t (e_k - e_m) dt \quad (2)$$

Using the trapezoidal rule of integration:

$$i_{k,m}(t) = \frac{\Delta}{2L} (e_k(t) - e_m(t)) + I_{k,m}(t - \Delta t) \quad (3)$$

Where the equivalent current source is known from the previous step.

$$I_{k,m}(t - \Delta t) = i_{k,m}(t) + \frac{\Delta t}{2L}(e_k(t) - e_m(t)) - e_m(t - \Delta t) \quad (4)$$

The expression (4) provides the equivalent impedance network of inductance (Fig. 1b). the same applies to mutual inductance, but instead of self inductance there is mutual inductance.

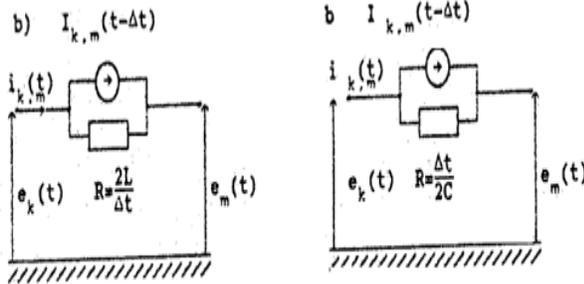


Fig.1-a. Inductance, b-Equivalent impedance network

Fig.2-a- Capacitance, b- Equivalent network

For the capacitance C between nodes K and m (Fig.2), the following expression holds:

$$i_{k,m}(t) = C \frac{d(e_k - e_m)}{dt} \quad (5)$$

To which the trapezoidal rule integration can be applied. 
$$\frac{i_{k,m}(t) + i_{k,m}(t - \Delta t)}{2} = C \frac{e_k(t) + e_k(t - \Delta t) - e_m(t) - e_m(t - \Delta t)}{2\Delta t} \quad (6)$$

If settled:

$$i_{k,m}(t) = \frac{2C}{\Delta t} [e_k(t) - e_m(t)] + I_{k,m}(t - \Delta t) \quad (7)$$

Then where:

is the equivalent current source from the previous step.

$$I_{k,m}(t - \Delta t) = -i_{k,m}(t - \Delta t) - \frac{2C}{\Delta t} [e_k(t - \Delta t) - e_m(t - \Delta t)] \quad (8)$$

The equivalent impedance network of capacitance is given in fig. 2b, and its form is identical to the one of the inductance. The same applied to mutual capacitances but instead of the capacitance there is mutual capacitance. The resistance is replaced with the expression ( Fig.3) and

$$i_{k,m}(t) = \frac{1}{R} (e_k(t) - e_m(t)) \quad (9)$$

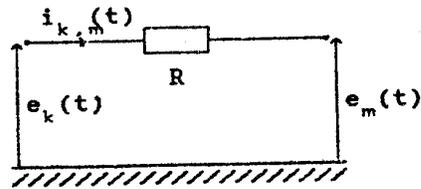


Fig. 3. Resistance

The conductivity is replaced with the expression (Fig.4) and

$$i_{k,m}(t) = G(e_k(t) - e_m(t))$$

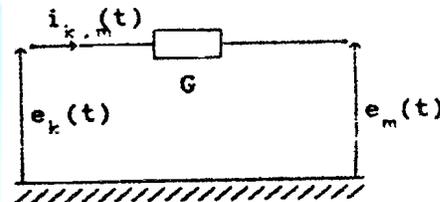


Fig. 4. Conductance

Let us refer to the equivalent diagram of a double-winding transformers in fig.5 that will serve as an example. The windings are divided in 4 elements, and their parameters are given. The unknown quantities in this case are the node voltages towards the earth ( u<sub>1</sub> u<sub>2</sub> and u<sub>3</sub>) and currents through the inductance(i<sub>1</sub>, i<sub>2</sub> and i<sub>3</sub>), while all the other parameters are known (C, L, R, G), because they are either determined from the transformer geometry, given in advance, or determined in the course of the computation. According to the Kirchoff's current law, the following matrix equations can be written:

$$\overline{\sigma 1}(t) = \overline{K} \frac{d}{dt} \overline{u}(t) - \overline{EK} \frac{d}{dt} U_o(t) + \overline{G}(t) \overline{u}(t) - \overline{EG}(t) U_o(t) \quad (10)$$

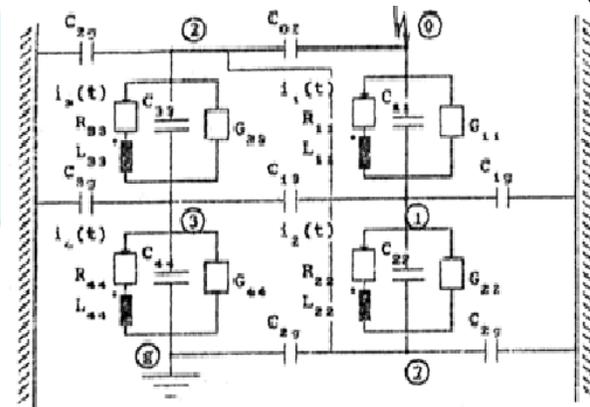


Fig. 5 Equivalent circuit

And, if generalized, i.e. if the network contains any number of nodes (let us designate them with 1) or any number of elements (let us designate them with m), we can

formulate the rule for formation of matrices according to Fig. 5. Formulation of matrices: is the connection matrix of inductances, it is formed so that the value of elements is +1 if the end of the j-the inductance is connected to the i-the node( the beginning of the inductance in the figure is marked with a point, and it depends on the windings direction); -1 if it is connected with the beginning, while other elements are 0.

$-\overline{i(t)}$  is the column vectors of unknown currents through inductances (n rows)

$-\overline{k} = \sum_{i=1}^n k_{ii} \sum_{j=1}^n k_{ij}$  is square matrix, where  $k_{ij}$  is capacitance between the i-the and the j-the node with negative sign, and  $K_{ii}$  is the sum of capacitances connected to the i-the node

$-\overline{u(t)}$  is the column vector of unknown voltages towards the earth (1 rows)

$-\overline{EK}$  is the column vector of unknown capacitances between the initial node (node 0) and the i-the node (1 row)

$-\overline{G(t)}$  is the conductance matrix, and is formed analogously to the matrix  $\overline{K}$  (dimension 1\*1)

$-\overline{EG(t)}$  is the column vector of connected conductance's between the initial node and the i-the node with I rows (analogously to the vector  $\overline{EK}$ )

All together, there are 1+n unknown quantities, so that it is necessary to write n equations (obtained through applications of Kirchhoff's voltage law). Hence, according to Fig. 5, the equations can be written in matrix from as follows:

$$\overline{F} U_0(t) - \overline{D} \overline{u(t)} = L \frac{d}{dt} \overline{i(t)} + \overline{R(t)} \quad (11)$$

Formation of matrices:  $-\overline{F}$  Is the column connection vector of initial voltage. It is in face negatively transposed row of initial node matrix  $\overline{D}$

$$-\overline{D} \text{ is the transposed matrix } \overline{D} \quad -\overline{L} = \sum_{i=1}^n l_{ij} \quad (12)$$

$l_{ij}$  is the inductance square matrix,  $l_{ii}$  is the natural inductance of the elements I, while  $l_{ij}$  is the self inductance between i-th and the j-th element

$$-\overline{R(t)} \sum_{j=1}^n R_{ij}$$

is the resistance square matrix with diagonal elements only. If the trapezoidal rule of integration is applied to linear differential equations (11) and (12), it results:

$$\frac{1}{2} [\overline{D} \overline{i(t)} + \overline{D} \overline{i(t - \Delta t)}] = \overline{k} \overline{u(t)} \frac{1}{\Delta t} - \overline{k} \overline{u(t - \Delta t)} \frac{1}{\Delta t} - \overline{EK} U_0(t) \frac{1}{\Delta t} + \overline{EK} U_0(t - \Delta t) \frac{1}{\Delta t}$$

$$\frac{1}{2} [\overline{F} U_0(t) + \overline{F} U_0(t - \Delta t) - \overline{D} U(t) - \overline{D} \overline{u(t - \Delta t)}] = L \overline{i(t)} \frac{1}{\Delta t} - L \overline{i(t - \Delta t)} \frac{1}{\Delta t} + \overline{R(t)} \overline{i(t)} \quad (13)$$

The solving of equation system (13) and (14) provides unknown voltages of nodes towards the earth at the moment

$$\overline{u(t)} = \frac{-1}{Y(t)} \left[ \left[ \frac{1}{2} \overline{P(t)} + 2\overline{EK} \frac{1}{\Delta t} + \overline{EG(t)} \right] U_0(t) + \left( \frac{1}{2} \overline{P(t)} - 2\overline{EK} \frac{1}{\Delta t} \right) + \frac{U_0(t - \Delta t)}{\overline{D}} + \left( \frac{-1}{2} \overline{V(t)} + 2\overline{k} \frac{1}{\Delta t} \right) \overline{u(t - \Delta t)} + \left( \overline{T(t)} \frac{1}{\Delta t} + \overline{D} \right) \overline{i(t - \Delta t)} \right] \quad (14)$$

Where

$$\overline{S(t)} = L \frac{1}{\Delta t} \overline{R_{-s}(t)} \quad \overline{P(t)} = \frac{-1}{DS(t)} \overline{F} \quad \overline{T(t)} = \frac{-1}{DS(t)} L \quad \overline{V(t)} = \overline{D} \overline{S(t)} \overline{D} \quad \overline{Y(t)} = \frac{1}{2} \overline{V(t)} + 2\overline{K} \frac{1}{\Delta t} + 2\overline{G(t)} \quad (15)$$

$\overline{u(t - \Delta t)}$  vector of the voltage towards the earth in the previous step  $U(t)$  input voltage: impulse

$$U_0(t) = A(e^{st} - e^{s_1 t})$$

Or sinusoidal damped

$$U_0(t) = A e^{-\zeta t} \sin(\omega t + \varphi)$$

$\Delta t$  step is computation

$\overline{Y(t)}$  square matrix of node conductance, 1\*1 and current through inductance at the moment t

$$\overline{i(t)} = \frac{1}{2} (\overline{R1(t)} (U_0(t) + U_0(t - \Delta t)) - \overline{R2(t)} [\overline{u(t)} - \overline{u(t - \Delta t)}] + \overline{R3(t)} \overline{i(t - \Delta t)}) \frac{1}{\Delta t} \quad (16)$$

Where

$$\overline{R2(t)} = \frac{-1}{s(t)} \overline{D} \quad \overline{R3(t)} = \frac{-1}{s(t)} \overline{L}$$

$$\overline{R1(t)} = \frac{-1}{s(t)} \overline{F}$$

$\overline{i(t - \Delta t)}$  vector of current through inductance in the previous step. since the conductance's in the matrix G and the resistances in the matrix R are variable in time, it is necessary to compute all the matrices of the systems (2.15) and (2.16) in every step. The elements of matrices G and R are computed in the moment t on the basis of the three previous steps, so that it is not necessary to exclude them as non-linear elements.

Connecting non-linear elements into a network If a branch between nodes K and m containing a non-linear element is taken, then, according to the compensation theorem, this element can be excluded from the network (matrix Y) and simulated through the current source i. for the nodes K and m according to the Thevenin theorem, the equation can be derived from the linear system (index 0 refer to the solution without the non-linear branch). The second equation is obtained from the ZnO characteristics that is non-linear

$$u_R(t) - u_m(t) = BRO \sqrt{\frac{i_{k,m}(t)}{c}} \tag{18}$$

where BRO is the number of series-connected arresters elements. The constants --- and c are obtained so that from the arresters characteristic two points for the voltage and the current are inserted in the equation (18), what results in two equations whose solutions yield a and c.

The Thevenin impedance z is obtained by multiplying the network impedance y(t) in equation (2.15) with the vector having elements o, except in the case of k-th nod with +1 and m-th mode with -1; in the case that node m is connected to the input voltage or the earth ,its value is 0.If presented mathematically

$$\overline{z} \overline{y}(t) \begin{cases} +1 \text{ for } k - \text{th node} \\ 0, \text{ except} \\ -1 \text{ for } m - \text{th node} \end{cases}$$

Where z is the vector of Thevenin impedance between the nodes k and m – in fact ,it is the difference between the columns k and m in the matrix y, Zt =Zk - Zm, for solving the non-linear equation (2.17) and the linear one (2.18) the Newton - Raphson method was chosen , because it provides the solution in only several steps .

In short, the solving process is as follows:

1. to calculate from equations (15) and (18) the equivalent Thevenin impedances between the nodes with arresters.
2. to calculate the voltage nodes u(t) for the moment t without the arresters.
3. to calculate the scalar equations (17) and (18) using the Newton – Raphson method (see fig. 6).
4. the final solution is obtained through superposition of u (t) =u (t) (0)-z I k, m

$$u(t) = u(t)^{(0)} - \overline{z} i_{k,m}$$

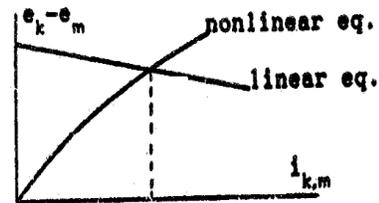


Fig. 6. Simultaneous solution of two equations

The superposition is possible only if performed on the linear part of the network.

### III.RESULTS ANS DISCUSSION

As shown above we have performed a computation of overvoltage's on one regulating transformer 60 MVA and 150 KV. The input voltage is the standard Impulse surge 1.2/50, the amplitude 500 KV , and the oscillogrms of voltages and currents on the regulating winding are given in figures 8 and 9.

The curve a in fig. 8b, shows the voltage shape with znO arrester (reference voltage u = 52 KV - the voltage at 1 mA d.c. ). The shape of the current through the arrester is given in fig . 9a, as well as the amount of absorbed energy . Through comparison of voltage shapes with and without ZnO arrester it can be seen how the arrester limits the voltage

The similar applies if the reference voltage is even lower (39KV) , fig. 8 c , It is logical that the current (fig. 9b) and the absorbed energy have been increased .

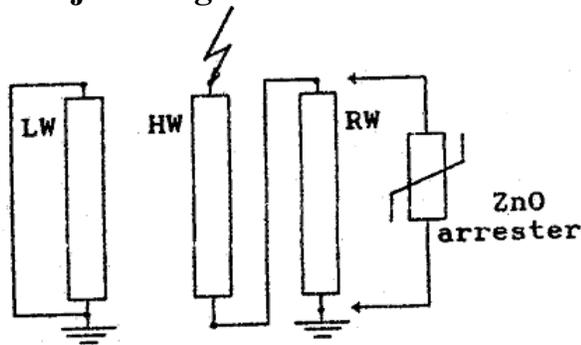


Fig. 7. Connection diagram

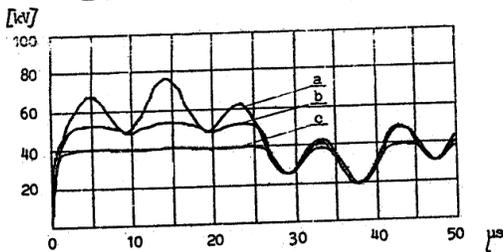


Fig. 8. Voltage across tap windings (through ZnO arrester)

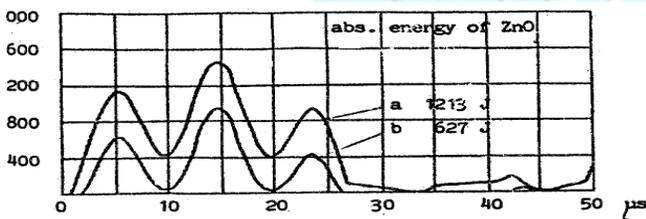


Fig. 9. Current through ZnO Arrester

IV. CONCLUSION

After implementation of the program, the system give a way for computation of overvoltage's between windings of transformers with Zn0 arresters, with the possibility of giving the oscillatory wave besides the impulse one.

Through the oscillatory wave it is possible to simulate service conditions and various disturbances in the network, the computation results are possible to used for dimensioning the transformers insulation and lightning arresters of power transformers.

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